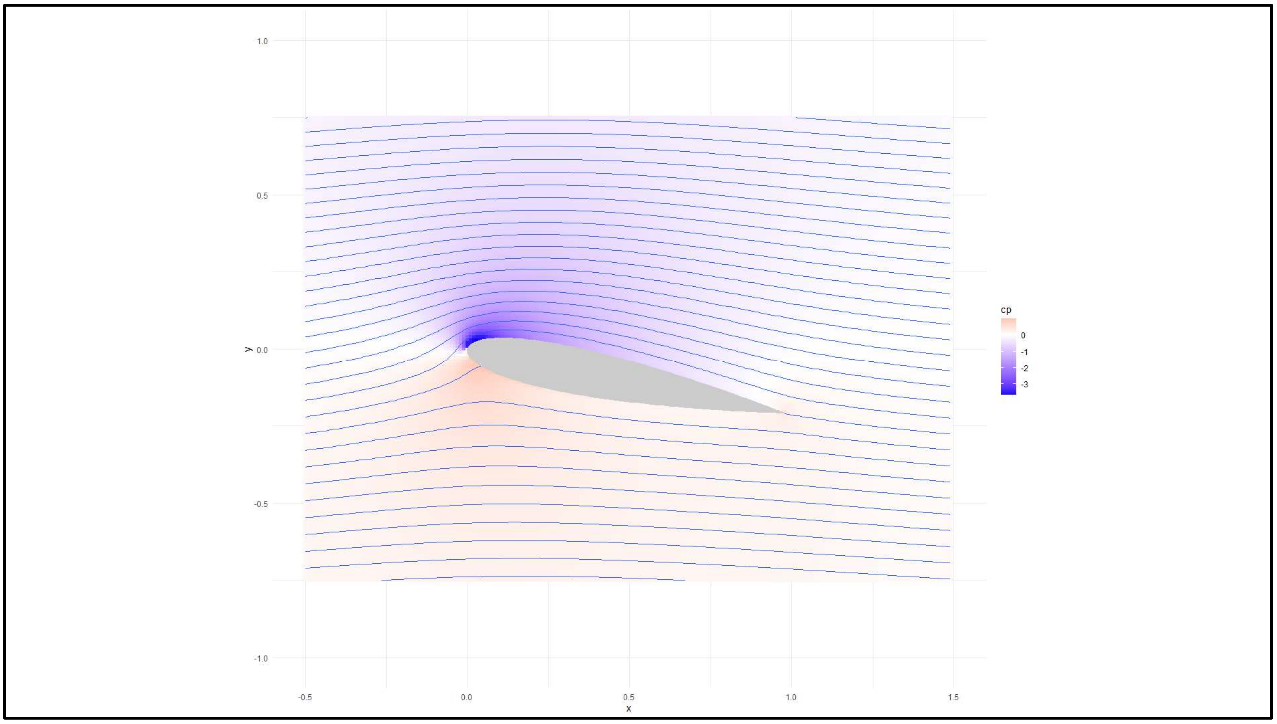
It’s a Drag—Part 2

induced drag and its friends
(*wake turbulence, ground effect,
trim drag, winglets)*



Lift arises from the difference in pressure between the top of the airfoil (where the pressure is less than ambient) and the bottom (where the pressure is higher than ambient).

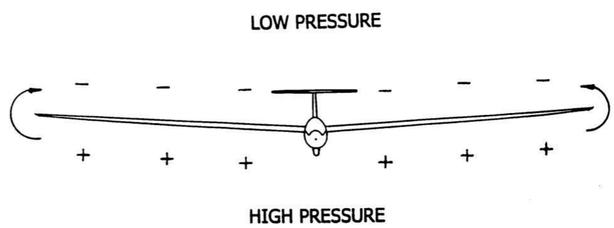
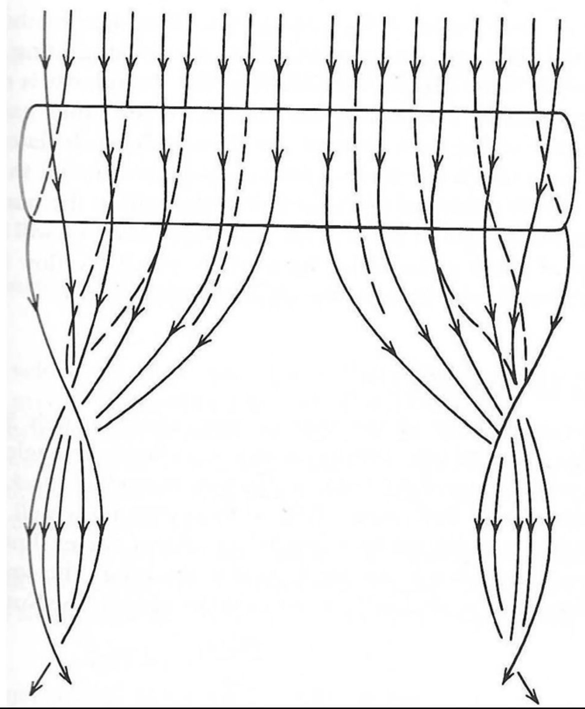
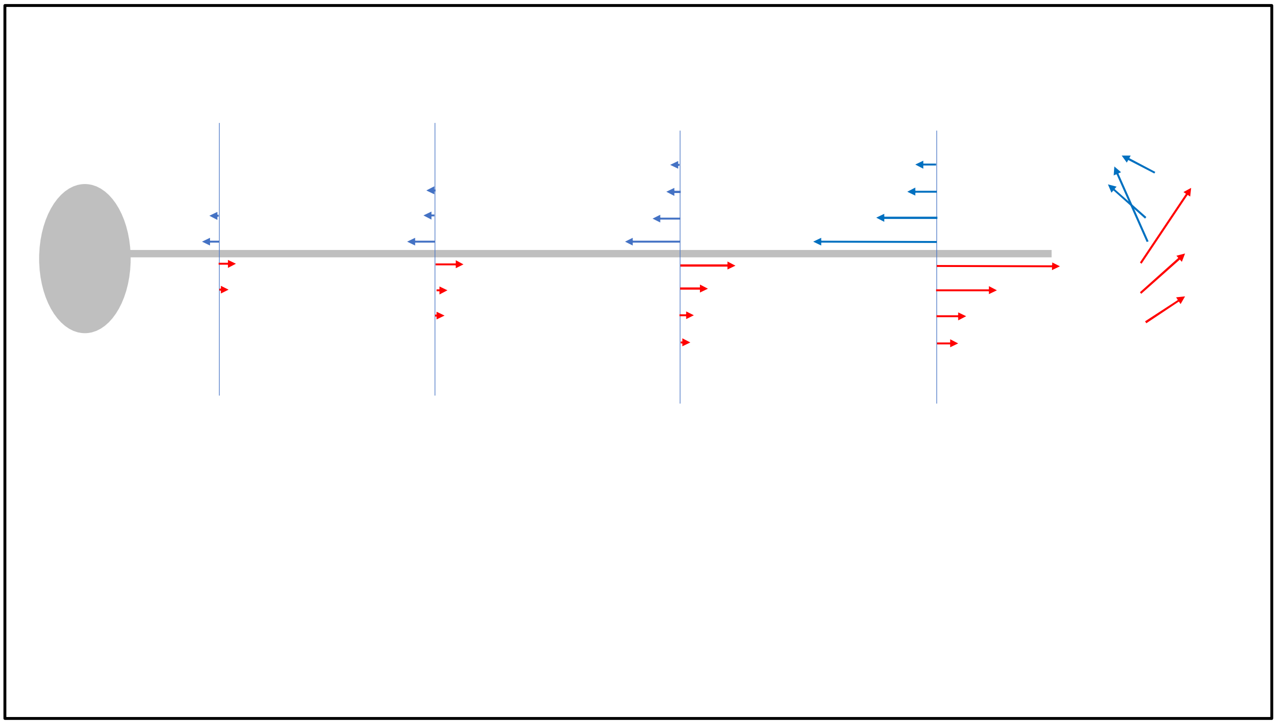


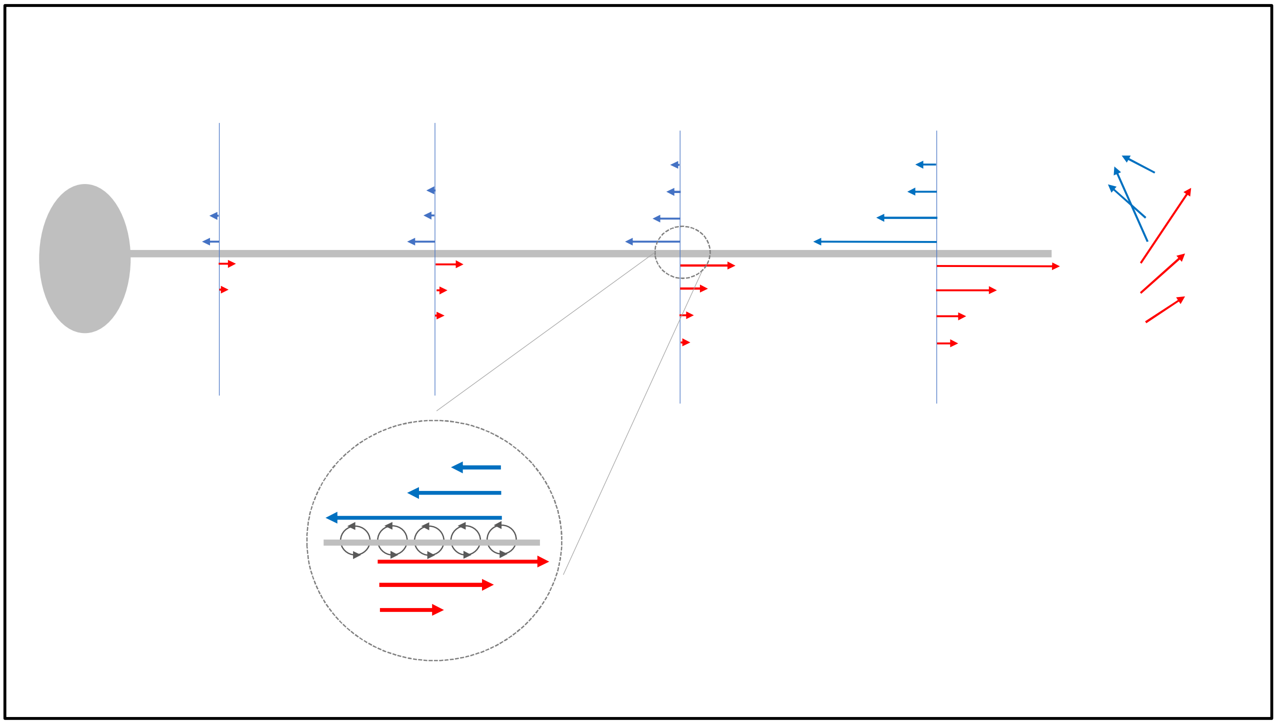
Fig. 2 Higher pressure air on the wing lower surface flowing around wingtip to upper surface.



The flow around the wingtip displaces the streamlines towards the wingtip on the lower surface (dashed) and away from the wingtip on the upper surface (solid) . Combined with the freestream velocity, this spanwise flow creates a swirling motion downstream of the wing, resulting in two trailing vortices. Ley’s note that, while these vortices are commonly referred to as “wingtip vortices,” they origin spans the entire wingspan.

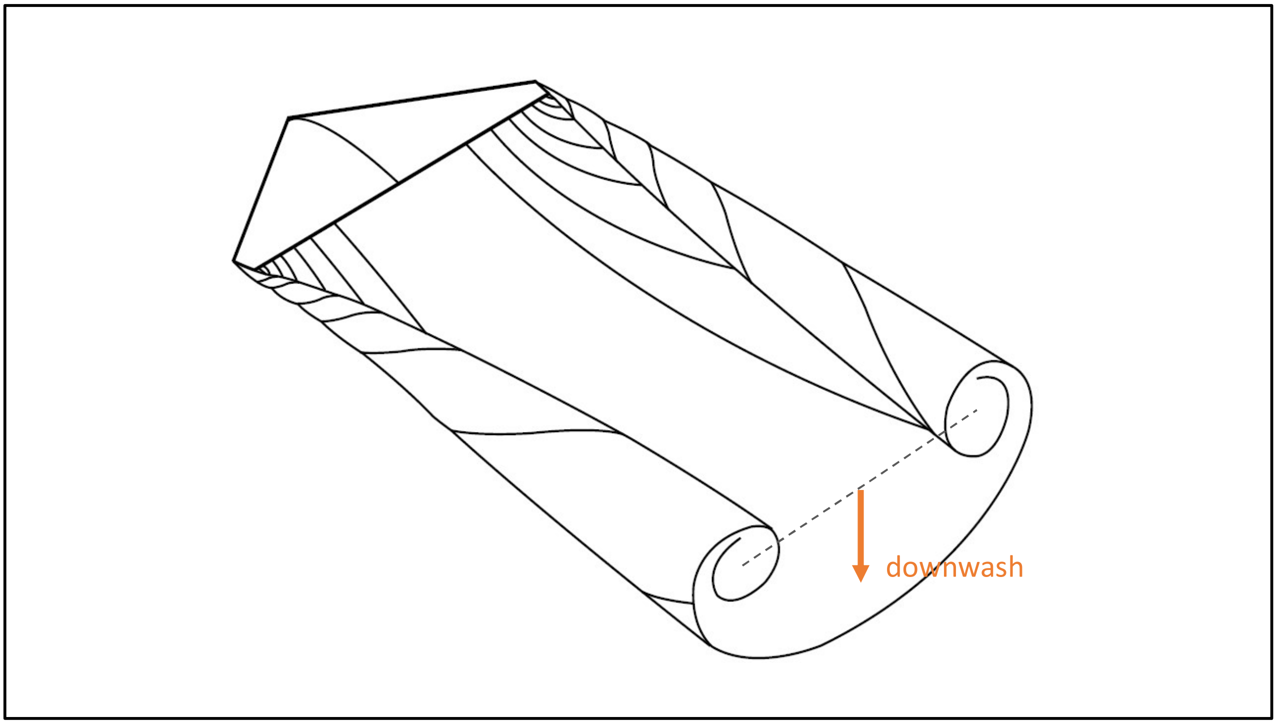


View from behind the trailing edge, looking forward (fuselage on the left, wingtip on the right). Red arrows show the spanwise flow (crossflow) on the lower surface, blue on the upper surface. A shear layer is present at the trailing edge.



Another way to represent a shear layer is as a “vortex sheet”. We can imagine the shear layer being generated by a large number of tiny vortices leaving the trailing edge in the longitudinal direction (perpendicular to the view above). Considering two adjacent vortices, the vertical velocity components at the trailing edge cancel each other, leaving only the horizontal components. The horizontal velocity is higher closer to the vortex axis and diminishes rapidly with increasing distance.

A characteristic of such a vortex sheet is that it will “roll-up” into macroscopic vortices, as illustrated in the next slide.



So, we can imagine the flow field behind the wing as consisting of a vortex sheet shed from the trailing edge, rolling-up further downstream into two large scale vortices—the wake vortices responsible for the dreaded "wake turbulence." Let's note that the clockwise (looking in the direction of flight) rotational flow field associated with the left vortex will tend to push the right vortex down, and the counterclockwise rotation attached to the right vortex tends to push the left vortex down, resulting in the wake vortices sinking behind the wing. In between the vortices, the flow velocity has a downward component—the "downwash." As we will see later, the presence of the downwash reduces the wing angle of attack (hence reducing lift) and tilts the lift rearwards, generating a drag component called "induced drag."

Frederick Lanchester

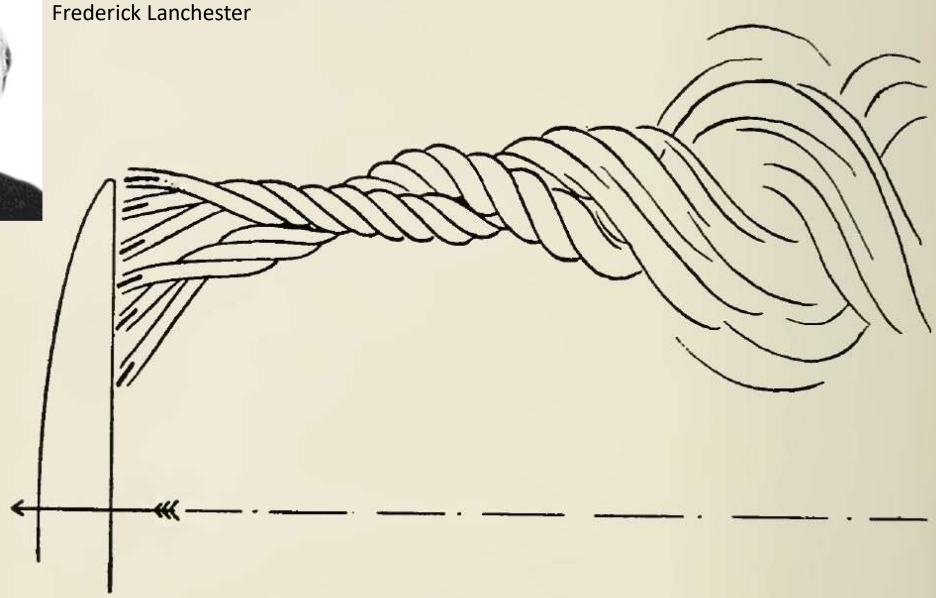
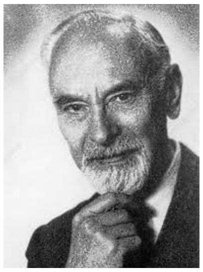


Fig. 86.

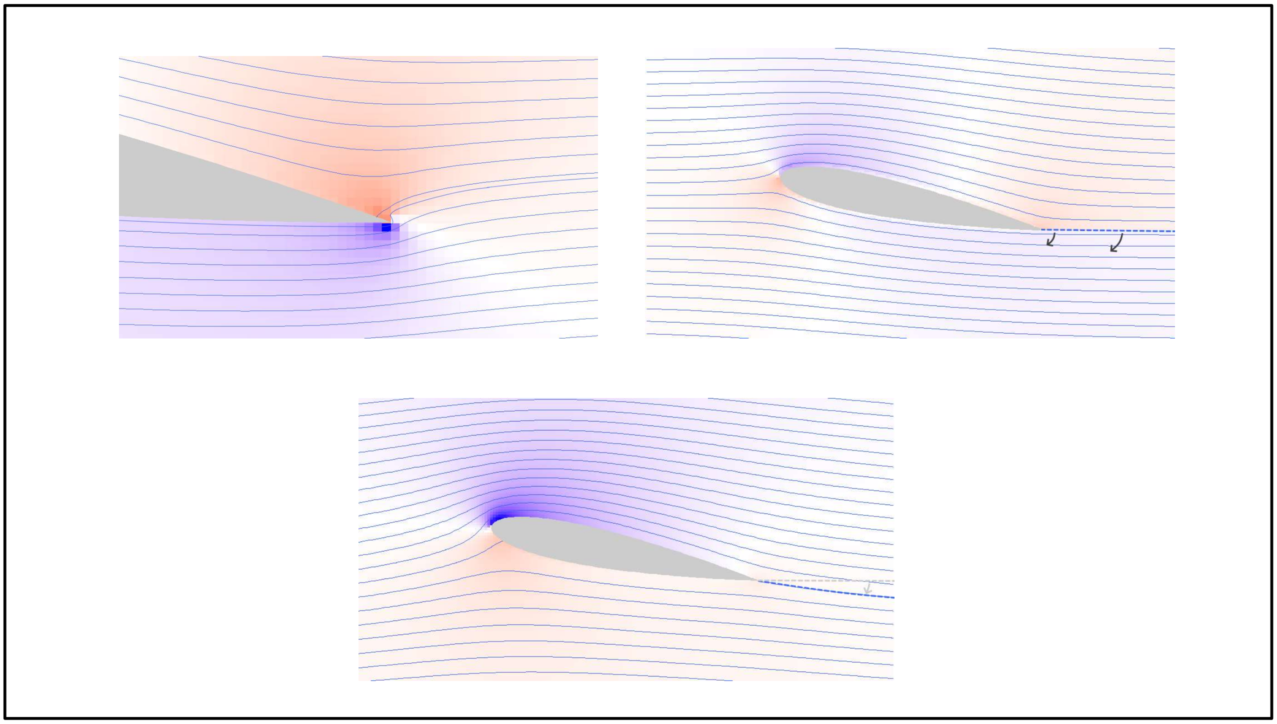
The English polymath and engineer Frederick Lanchester (https://en.wikipedia.org/wiki/Frederick_W._Lanchester) was the first to imagine the flow configuration behind the wing, and to associate it with a component of drag. As shown in his diagram above, trailing vortices are shed from the wing trailing edge, and roll-up into wake vortices.



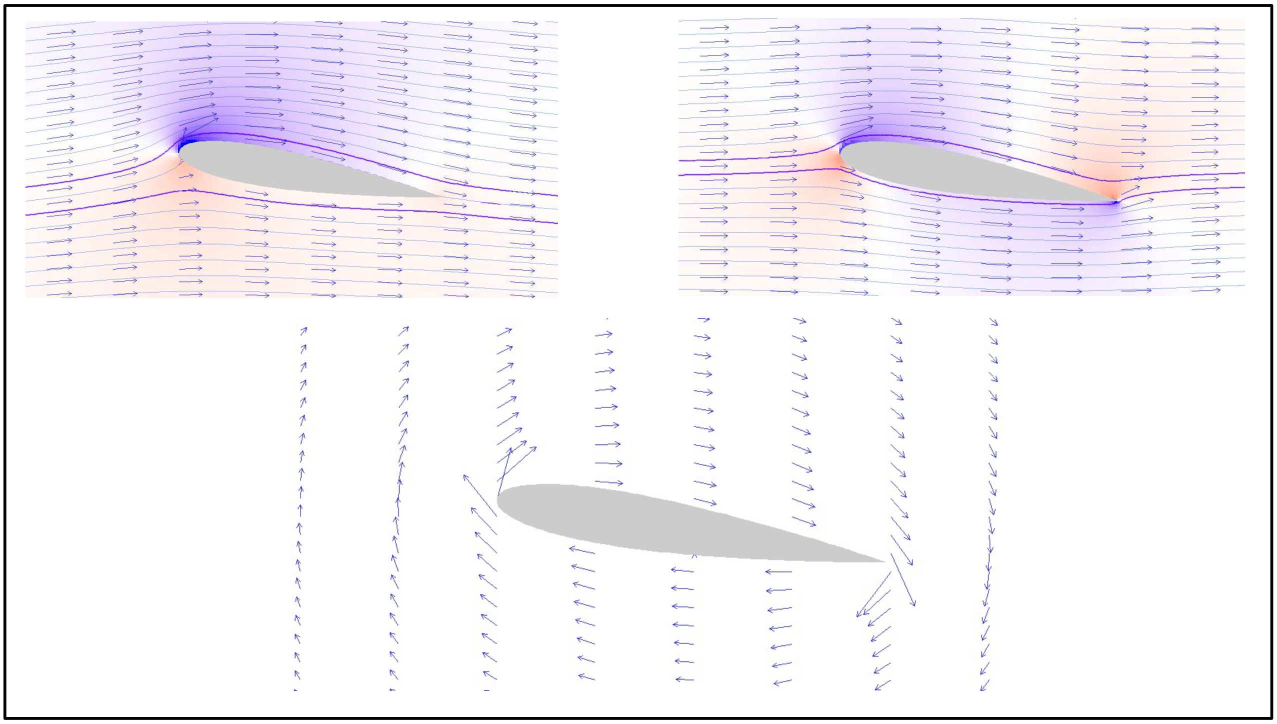
Ludwig Prandtl

Prandtl's Lifting Line Theory for Induced Drag

A simplified model for the flow around a wing of finite span

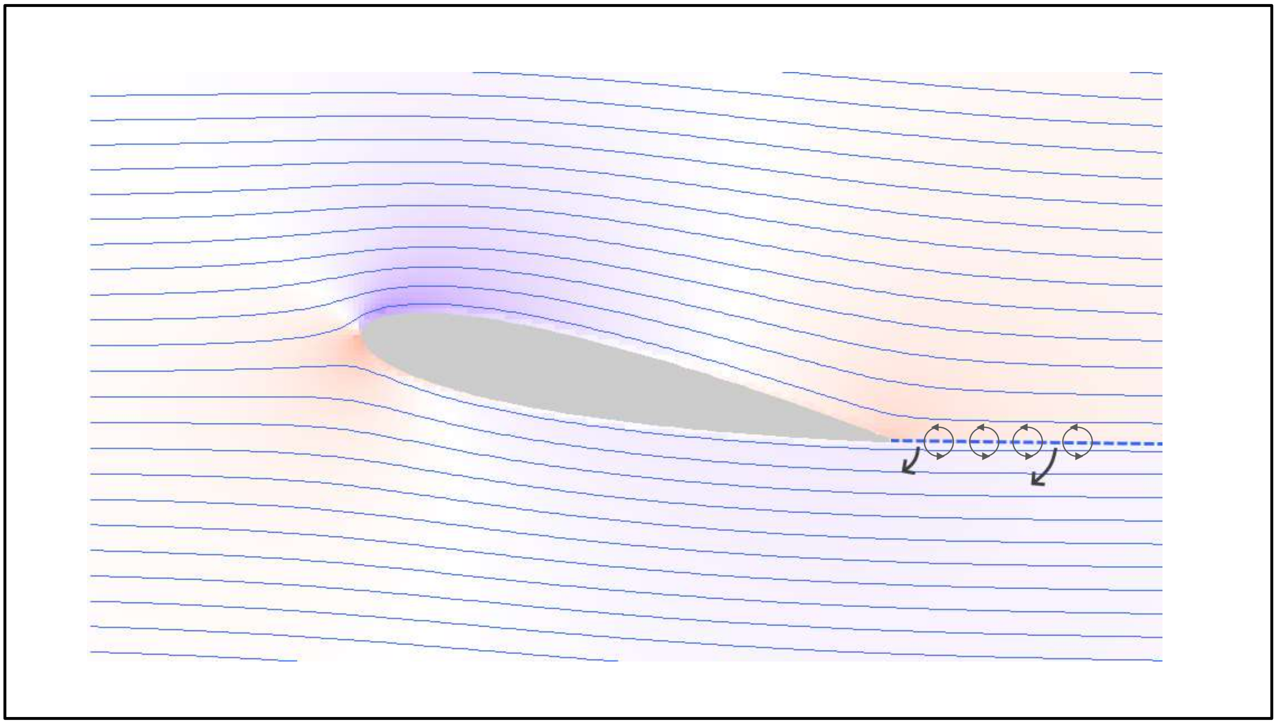


First, let’s review the mechanism of lift generation. The top left illustration shows the flow around the trailing edge resulting from inviscid flow theory (Euler equations)—which predicts zero drag and zero lift, a result contradicted by experience. However, the flow is shown to make a sharp turn around the trailing edge. Any real fluid (like air) has some viscosity, and, due to the development of a boundary layer, the flow from the lower side of the airfoil will in fact separate at the trailing edge instead of making that sharp turn, and proceed straight in the downstream direction, as illustrated on the top right. The dashed line indicates the surface separating the flows from the upper and lower sides. We’ll note that the pressure just above the separation surface is higher than ambient, and lower than ambient just below. According to Bernoulli’s law the velocity just above the surface will thus be lower above than below, and the separation surface is also a shear layer. Under the effect of the pressure difference, the separation surface will migrate downwards, eventually reaching the configuration shown in the bottom illustration. In that configuration, the flow smoothly exits the trailing edge in the direction of the mean camber line—this is known as the “Kutta condition.” That new flow configuration results in a “bending” of the flow around the airfoil in a downwards direction, decreasing the pressure over the upper surface and increasing the pressure over the lower surface, thus generating lift.

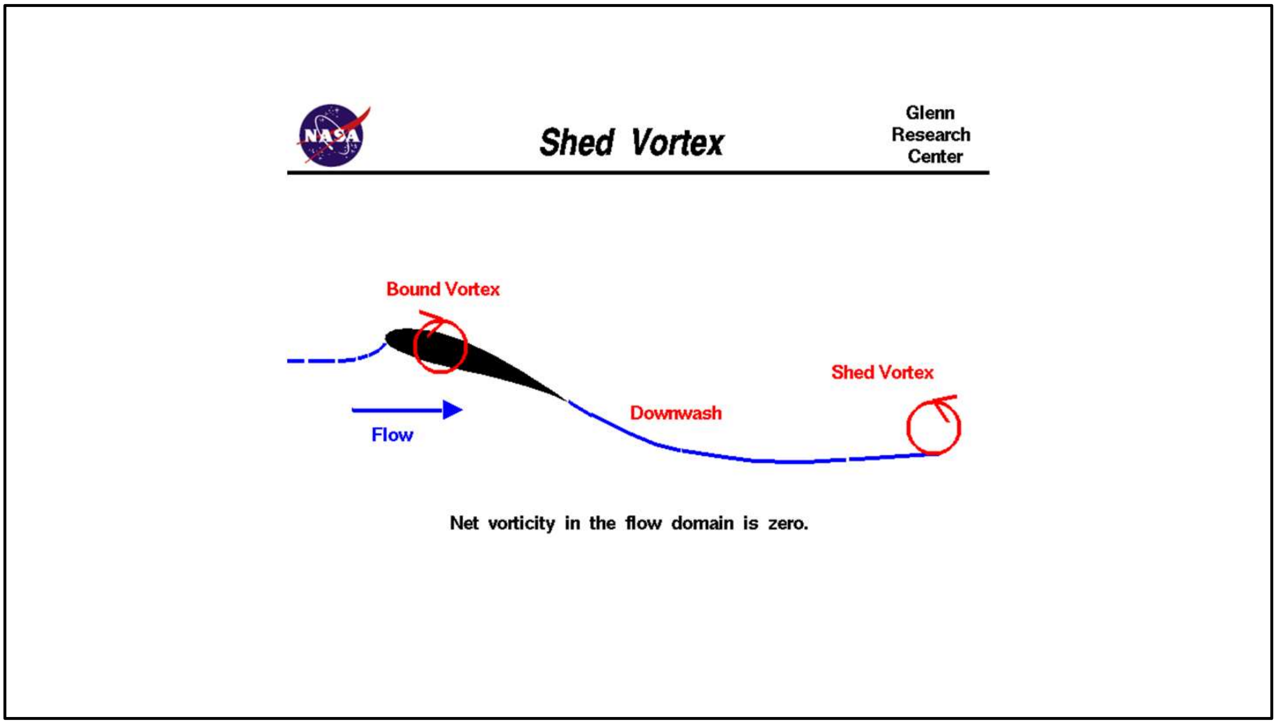


The top left diagram displays the flow velocity around the airfoil when the Kutta condition is met (i.e., when lift is being generated), while the top right one displays the theoretical velocity field predicted by the inviscid theory (no lift). The bottom one shows the difference between the two velocity fields (the scale has been increased for clarity). A striking feature is the vortex pattern. Kutta and Joukowski showed that this pattern is directly related to the generation of lift. We'll note that the clockwise ration around the airfoil increases the speed on the upper surface (hence lower pressure per Bernoulli), and decreases it on the lower one (hence higher pressure).

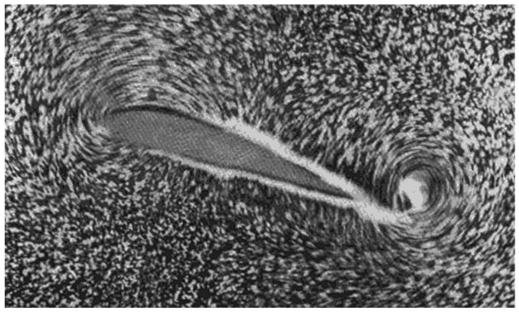
In the next few slides, we'll have a deeper look at the genesis of that vortex motion.



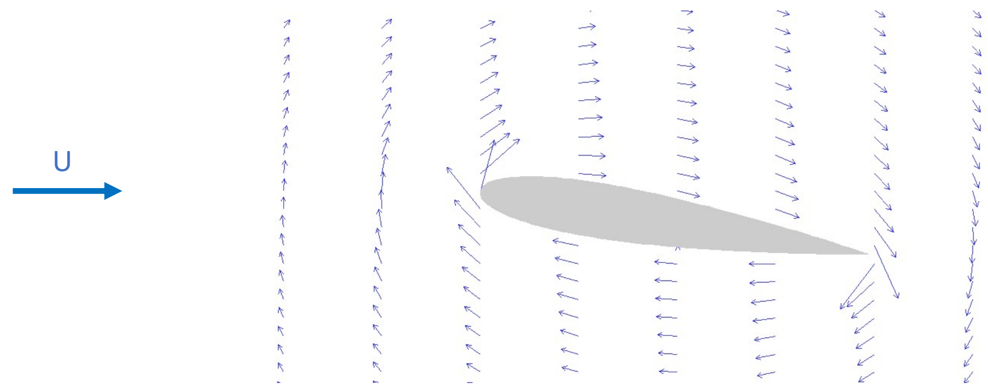
First, let’s note that the previously mentioned shear layer that develops upon starting the flow can be also be viewed as a vortex sheet—see rationale on slide no. 6.



The vortex sheet “rolls-up” into a counter-clockwise rotating vortex as illustrated above. This shed vortex, a.k.a. starting vortex, drifts downstream, entrained by the free stream velocity. Owing to an important theorem due to Helmholtz, a vortex rotating in the opposite direction (i.e., clockwise) will develop around the airfoil. This is known as the bound vortex--it is bounded to the airfoil and does not drift with the flow.

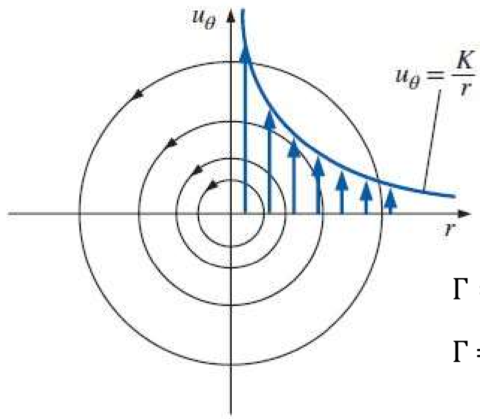


The starting vortex can be observed on the flow visualization above, depicting the flow around an airfoil moments after the flow is started.



vortex “strength” = circulation = Γ
Lift = ρ U Γ (Kutta-Joukovsky theorem)

The vortex motion we identified three slides ago, repeated above, is the incarnation of the “bound vortex.” Its strength is identified by a measurable quantity called “circulation” (Γ) which fundamentally is a measure of the intensity (speed) of the rotational motion. A remarkable result established by Kutta and Joukowski is that the lift generated by an airfoil is proportional to strength of the vortex, irrespectively of the details of the flow. As far as lift modeling is concerned, the airfoil can be replaced by a single vortex with a strength Γ such that the Kutta condition is met at the trailing edge of the airfoil.



$\Gamma = 2\pi r \ * \frac{K}{r} = 2\pi \text{ K} = cte$

Γ = circulation = vortex strength

And here is a representation of a single vortex. The flow rotates around the axis of a circular cylinder, with a speed inversely proportional to the radius—we can imagine it as a small tornado!

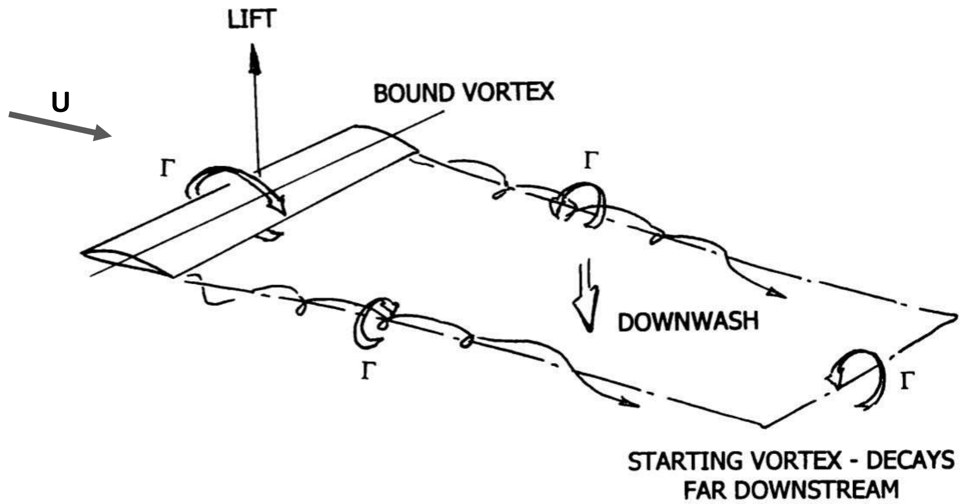
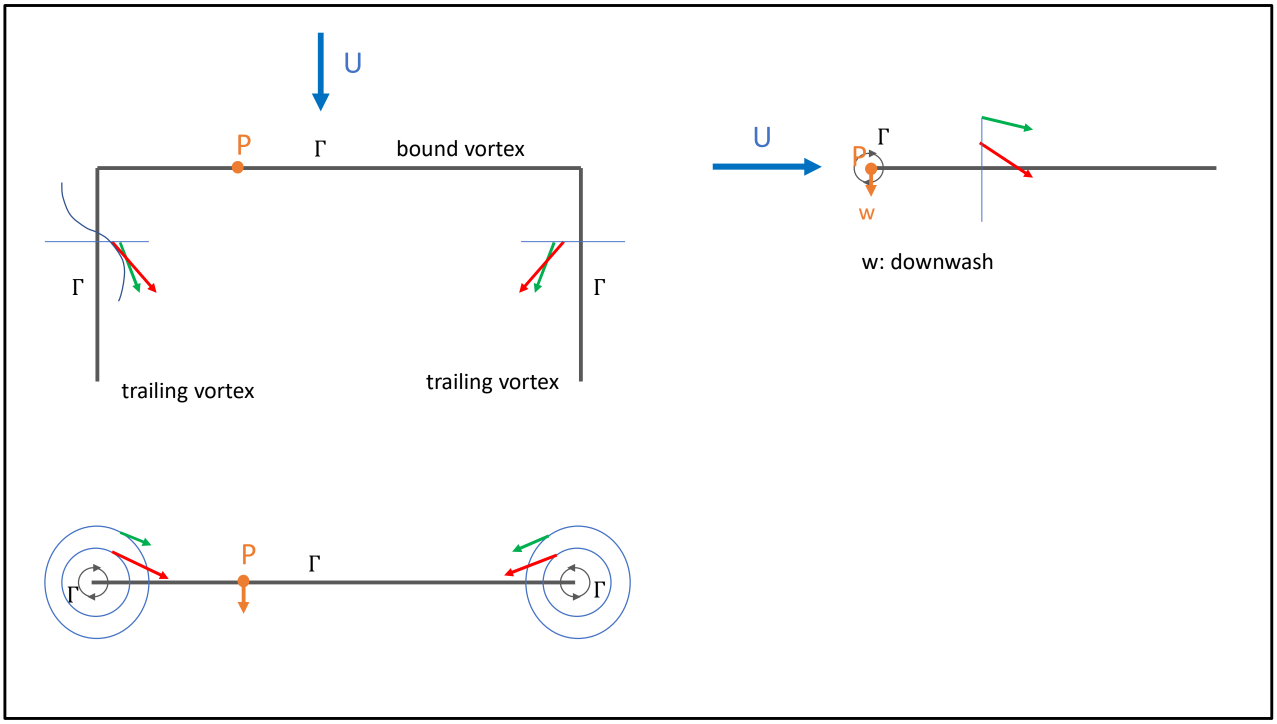


Fig. 4 Idealized "horseshoe" vortex system.

Other important results due to Helmholtz state that a vortex must have a constant strength along its length, and that vortices cannot just end in a flow field—they either have to extend to infinity or form a closed loop. This leads to this simplified model for the flow around a wing of finite span—a vortex loop of strength Γ embedded into a free stream of velocity U. Upon exiting the wing, the bound vortex becomes two trailing vortices connecting to the starting vortex, thus forming a loop. The starting vortex is advected to infinity, which means the model eventually simplified further to the bound vortex and two infinitely long trailing vortices. The counter-rotating trailing vortices induce a downward between the vortices—the downwash.

We'll note that this simplified model is consistent with the flow pattern described earlier based on physical considerations (trailing vortex sheet rolling-up into two counter-rotating vortices). This is encouraging! The advantage of the model above is that the vortex loop strength Γ can be calculated from the Kutta condition, and we can thus in principle calculate the flow field induced by the trailing vortices. In particular, we are interested in the value of the induced velocity at the wing position, for reasons that will become clear in the next few slides.



A three-view of the simplified model discussed in the previous slide, with a few selected velocities induced by the trailing vortices. Let's note that the velocity ***w*** induced at point P, located in the trailing vortices plane at the bound vortex (i.e., at the wing location), is pointing downward.

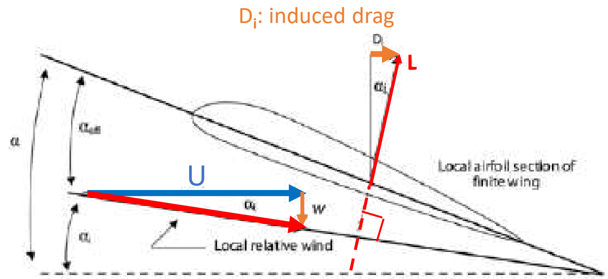
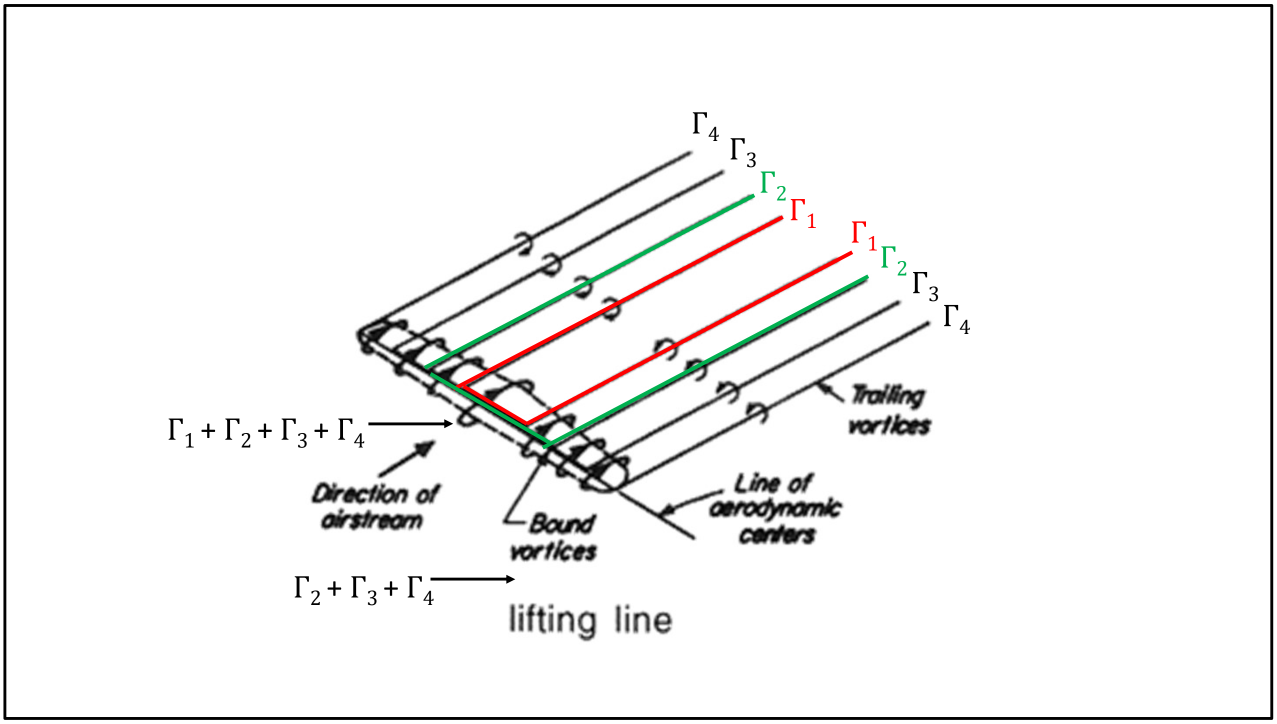


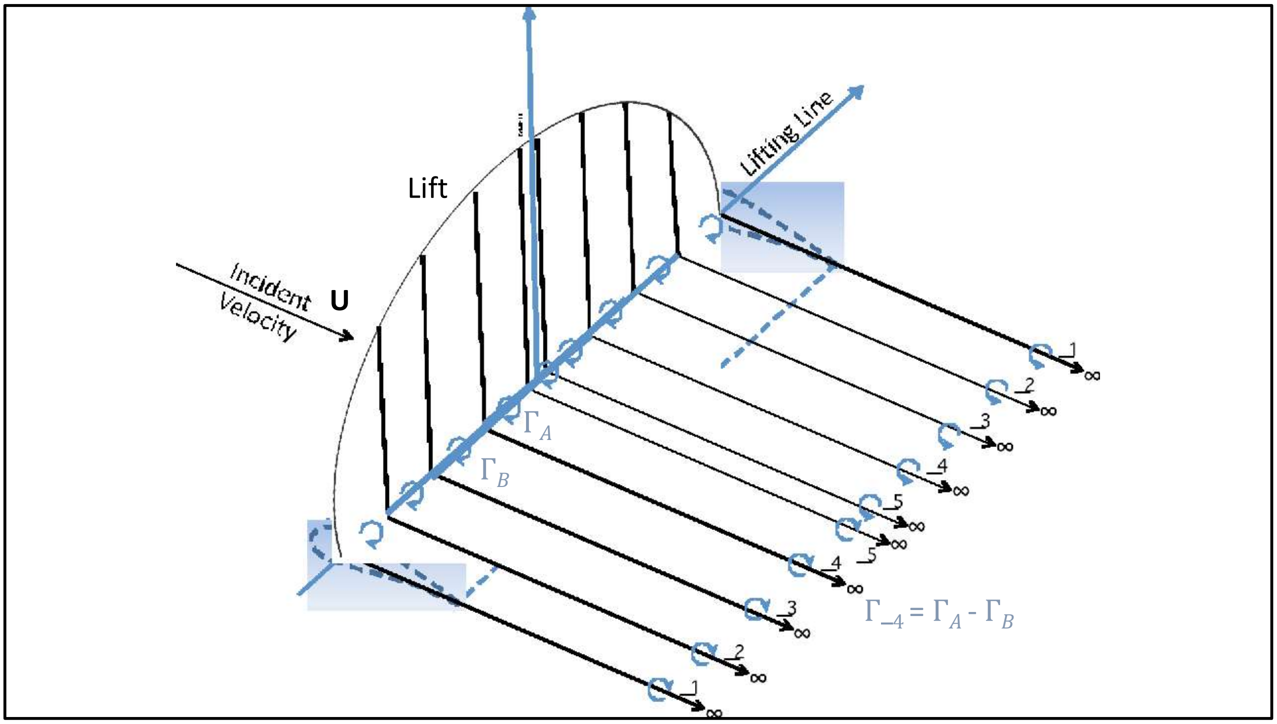
Figure 2. Effect of the downwash on a typical airfoil section of a finite wing.

In the two-dimensional case (infinite wingspan), lift is perpendicular to the free-stream velocity U, and no drag is being generated. In the case of a finite-span wing, the trailing vortices induce a downward velocity ***w***, as seen in the previous slide. Consequently, the local relative velocity the wing will experience is the velocity shown in red above. The corresponding lift will be perpendicular to this relative velocity (vector labeled L). This vector has a component Di lying along freestream velocity U. This force is opposing the motion of the aircraft—it is the drag component associated with the induced velocity, the so-called *induced drag*. Also, the angle of attack of the wing with respect to the local relative velocity is smaller than the one relative to the freestream velocity U. The lift will thus be reduced compared to the ideal two-dimensional case.

The power required to overcome this induced drag is Di * U, and, since energy is conserved, will be converted into the rotational motion of the wake vortices. So, the higher the induced drag, the more energy will be packed into the wake vortices responsible for wake turbulence.



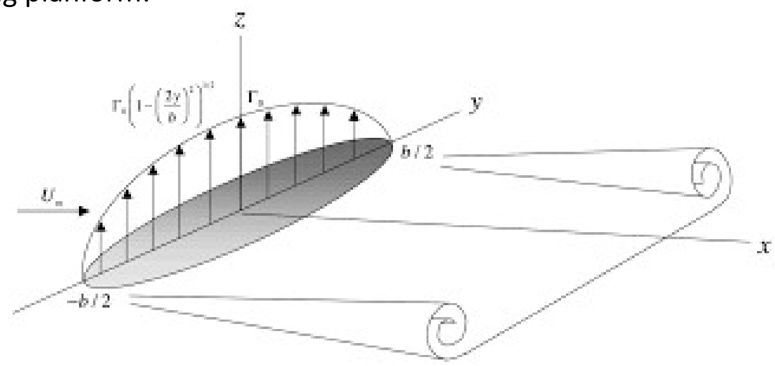
Now, the horseshoe vortex model is an oversimplification. It assumes that lift is constant spanwise until the wingtip is reached—but we just saw that, even in the case of a rectangular wing, the induced velocity will reduce lift. And in the general case of a non-rectangular wing planform, the lift (and thus circulation) will definitely be a function of spanwise location. This can be modeled as nested horseshoe vortices, as illustrated above.



The final simplified model for the flow around a wing of finite span, as developed by Prandtl, is shown above. It consists of a “lifting line”, a.k.a. the bound vortex representing the wing, and a vortex sheet made out of trailing vortices, the entire system being placed in a uniform flow with velocity U (the speed of the aircraft). The lift at any spanwise location equals the lifting line circulation at that location multiplied by the density and velocity U (Kutta-Joukowski theorem), and the strength of the trailing vortices is determined by the spanwise lift (and thus circulation) distribution.

However, the lift itself depends on the velocities (at the lifting line) induced by the trailing vortices… Expressing mathematically that the spanwise lift distribution is compatible with the strength of the trailing vortices results in an (integro-differential) equation for said spanwise lift distribution.

Elliptic wing planform:



$$D_i = \frac{1}{2}\rho v^2\, S\, C_{Di} \qquad \boxed{C_{Di} = \frac{C_L^2}{\pi A}}$$

Aspect ratio: A = $\frac{span}{mean\ chord}$ = $\frac{b^2}{S}$

The solution of the lifting line equation is pretty complicated in general, but simplified dramatically for an elliptic wing planform. In this case, in can be shown that (1) the induced velocity is constant along the span, (2) the lift distribution is also elliptical, (3) the induced drag is the minimum drag attainable for any planar wing * of a given aspect ratio, and (4) the induced drag coefficient is given by the formula displayed above, and depends only on the square of the lift coefficient and the wing aspect ratio.

(*) Better performance can be obtained with non-planar wings—see the discussion of winglets

Probably the most famous aircraft using the optimal, elliptically-shaped wing—the Spitfire.

Elliptical planform are not widely used--wings with straight leading and trailing edges are easier to fabricate, and with careful selection of spanwise twist and airfoil shape, can get close to the elliptical wing performance.

In general:

$$\boxed{C_{Di} = \frac{C_L^2}{\pi A\, e}}$$

Where is the spanwise efficiency, < 1 for non-elliptic planforms

For other planforms than elliptical, an efficiency ***e*** is introduced to account for the less-than-optimal induced drag performance.

$$C_{Di} = \frac{C_L^2}{\pi A e} \qquad A = \frac{b^2}{S}$$

b: wingspan
S: wing surface area

$$L = W = \frac{1}{2}\rho V^2 S C_L$$

$$\longrightarrow C_L = \frac{W}{\frac{1}{2}\rho V^2 S}$$

$$\longrightarrow C_{Di} = \frac{W^2}{\left(\frac{1}{2}\rho V^2\right)^2 S^2} \cdot \frac{S}{\pi b^2 e}$$

$$D_i = \frac{1}{2}\rho V^2 S C_{Di} = \cancel{\frac{1}{2}\rho V^2 S} \frac{W^2}{\left(\frac{1}{2}\rho V^2\right)^{\cancel{2}} S^{\cancel{2}}} \cdot \frac{\cancel{S}}{\pi b^2 e}$$

$$\boxed{D_i = \frac{2}{\pi e \rho} \left(\frac{W}{b}\right)^2 \frac{1}{V^2}}$$

$$\boxed{D_i = k \cdot \frac{W^2}{V^2}}$$

From the expression for the induced drag coefficient C_{Di}, we can deduce important results that impact our everyday flying.

Considering that the lift must be equal to the weight of the aircraft for straight-and-level flight, we can express the lift coefficient as a function of the aircraft weight and speed. We obtain that, for a given aircraft geometry, the induced drag is proportional to the square of the aircraft weight, and inversely proportional to the speed. So:

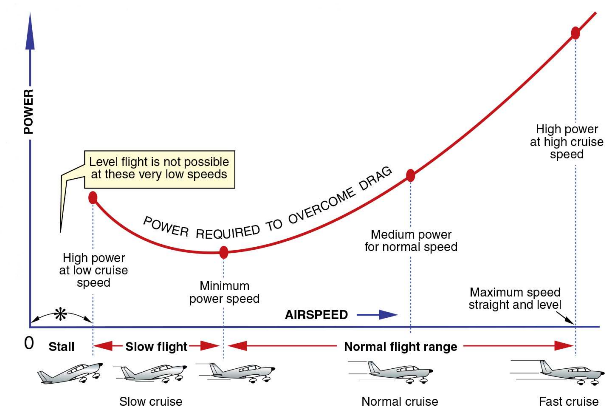
(1) Induced drag increases rapidly with decreasing airspeed
(2) The energy content of the wake vortices being a direct result of induced drag, wake turbulence intensity will increase rapidly with increased weight and with decreased airspeed. This outlines the fact that wake turbulence is at its most dangerous for heavy aircrafts flying at low speeds.

$$\boxed{\text{at } V_y ,\quad C_{D_o} \simeq C_{D_i} \quad D_o \simeq D_i}$$

$$D = \frac{1}{2}\rho V^2 S C_{D_o} + k\frac{W^2}{V^2}$$

$$= K_o V^2 + K_i \frac{1}{V^2}$$

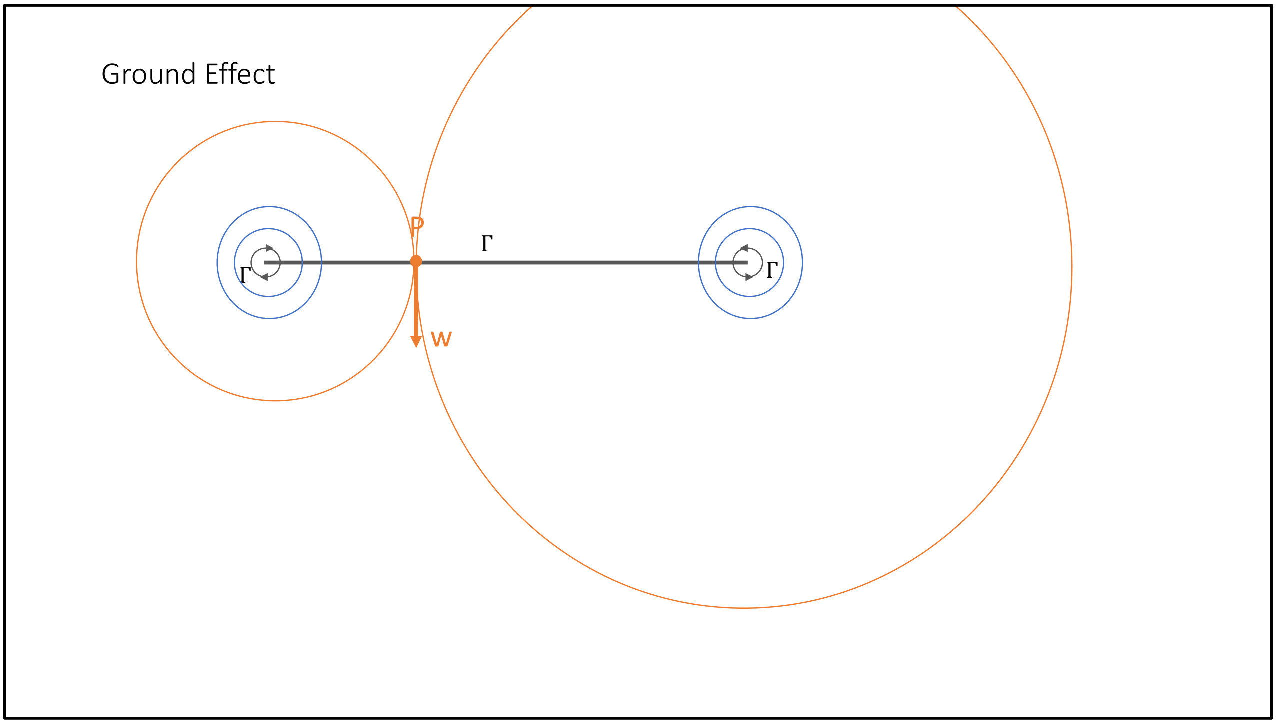
$$\text{Power} = D.V = K_o V^3 + K_i \frac{1}{V}$$



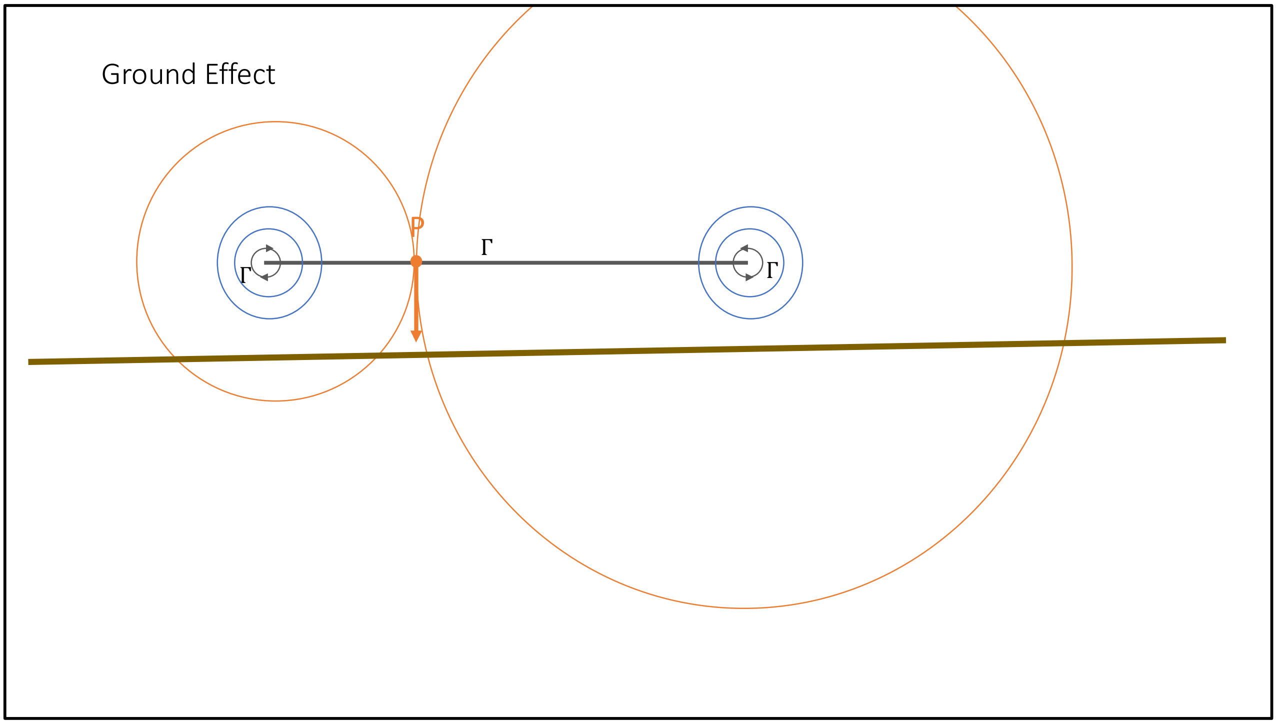
How does induced drag compare to parasite drag? A useful rule of thumb is that, at best rate of climb speed (***V_y***), induced drag is approximtey equal to parasite drag. At lower speeds, induced drag dominates, at lower speeds, parasite drag does.

The power required to compensate for drag, is Drag * Speed. Applying that to the total aircraft drag, we conclude that the power required for straight-and-level flight consist of a term proportional to the cube of airspeed (the parasite drag contribution), and a term proportional to the inverse of airspeed (the induced drag contribution). At higher speeds, the parasite drag contribution dominates, increasing as the third power of speed—going fast is expensive (a 10% increase in speed will roughly require a 30% increase in power—hence fuel flow!). At lower speeds though, the parasite drag contribution decreases rapidly, while the induced drag component increases. That’s why below a certain airspeed (roughly the best climb angle speed), the power required to maintain straight and level flight increases—we fly “behind the power curve.”

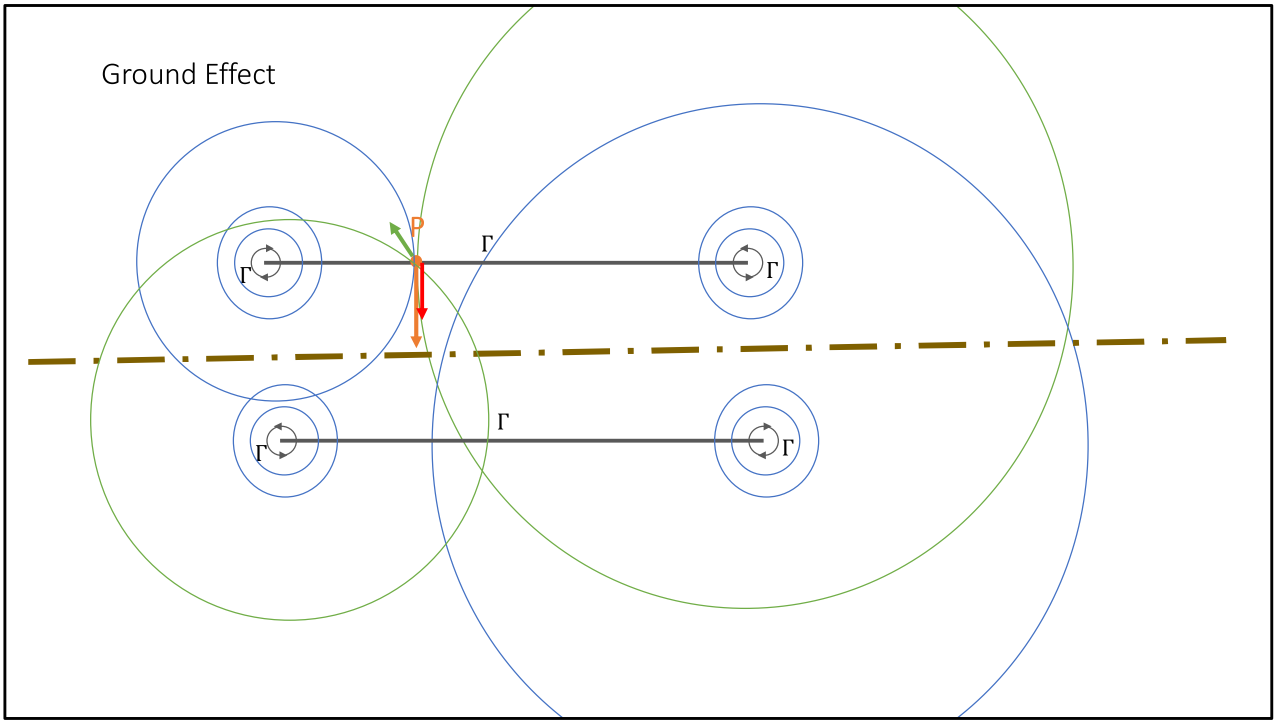
Why Ground Effect?



The above diagram illustrates the flow induced by the simplified “horseshoe” model discussed earlier. It presents a view from behind the trailing edge, looking forward. The induced velocity at spanwise location P, due to the right and left vortices, is represented as the orange vector w.



How can we model what happens if the wing flies in the proximity of the ground plane (brown line above)?

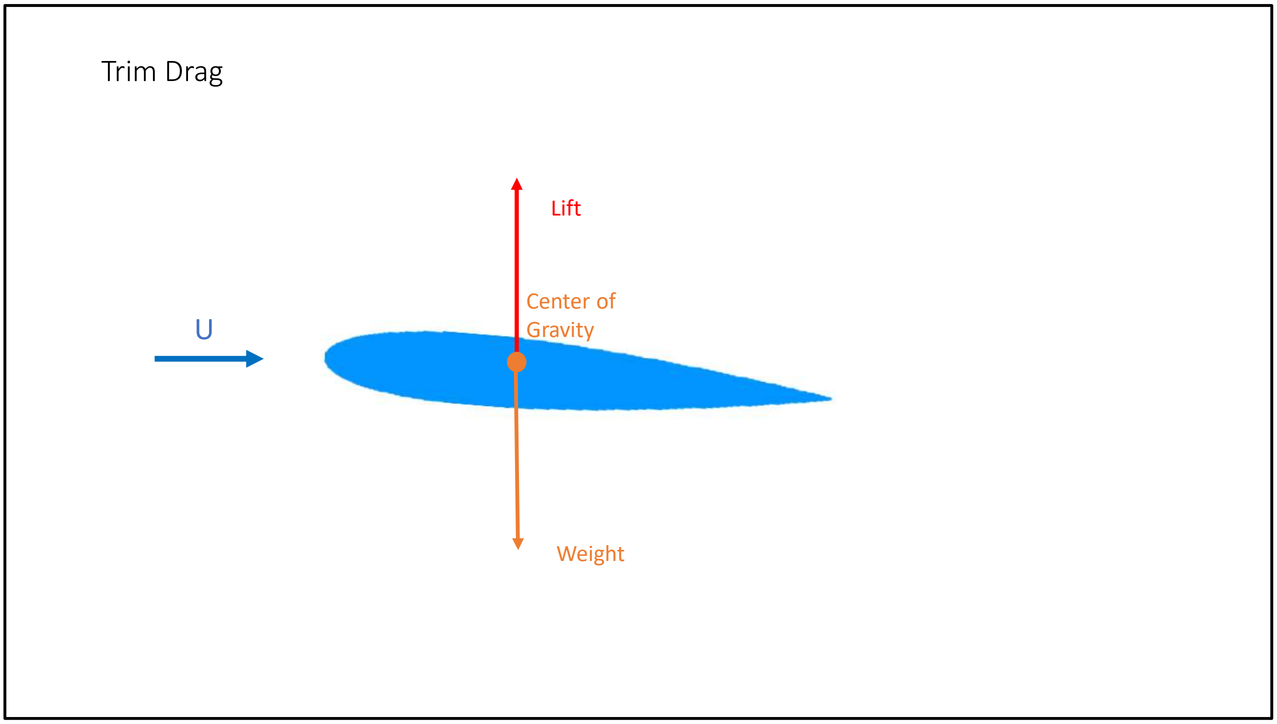


A useful trick is to place another “wing” (vortices system) symmetrically with respect to the ground plane, as shown above. By symmetry, no flow can penetrate the axis of symmetry, and the flow pattern in the upper half resulting from this configuration will thus be the same as the flow pattern in presence of the ground plane!

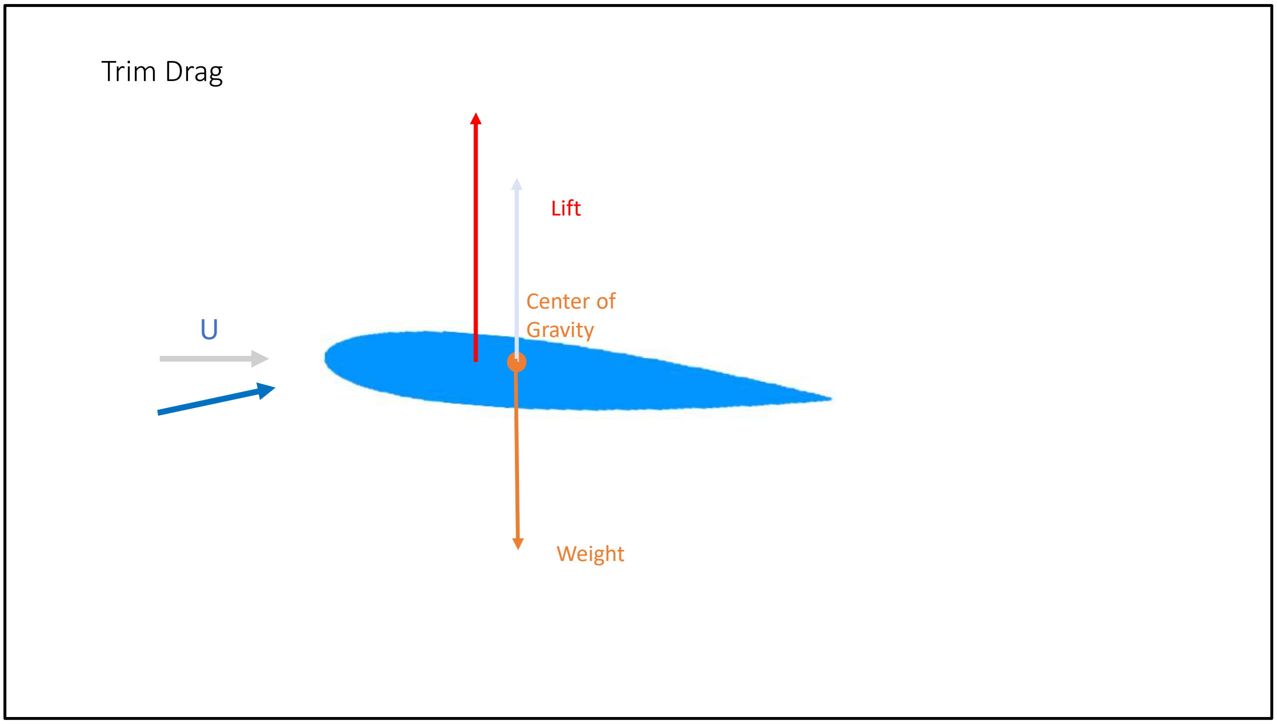
But in this case, the induced velocity at point P will be affected by the vortices in the lower half also. And these vortices induce a velocity (green arrow) that points generally upwards. The magnitude of the downwash (red arrow) for the combined system will thus be smaller than for the upper wing alone, reducing both the loss of lift and the induced drag. This effect will be noticeable only if the two “wings” are close enough for the velocities induced by the lower wing at P to be of meaningful magnitude.

This is why, when flying close to the ground, we experience increased performance and a tendency to “float”—the ground effect. This increase in performance is not actually a good thing in our everyday flying—when landing, it increases the landing distance, and when taking off, it might tempts us into trying to climb too aggressively!

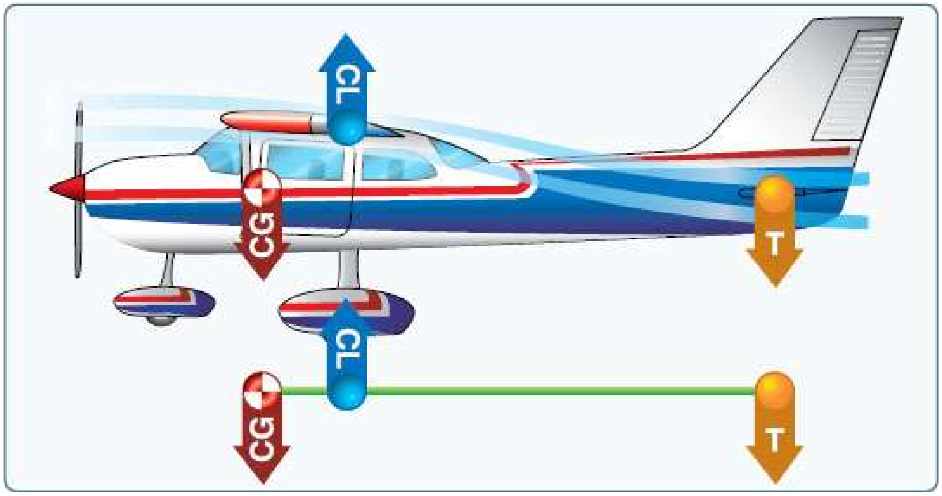
Trim Drag



Consider a single wing in straight-and-level flight. The weight acts through the center of gravity. For the wing to be in equilibrium, the lift vector must also be applied to the center of gravity.



Let’s imagine now that a gust momentarily increases the angle of attack. The lift will increase, but will also move forward (this is predicted by the Kutta-Joukowsky lift theory, and is verified in practice). This results in a moment that tends to *increase* the angle of attack even more—the system is unstable!

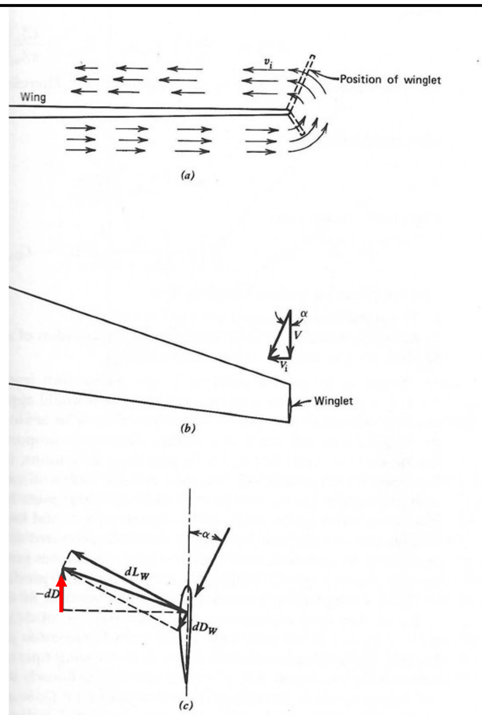


The traditional way to stabilize the system is to move the CG ahead of the center of lift of the wing, and add a stabilizer surface downstream of the wing, with a downward lift. As a result, the wing will have to provide additional lift (weight + T, vs. weight only). Consequently, the induced drag will be increased also. The more forward the CG is, the higher the induced drag penalty. The trim drag typically amounts to a few percent of total drag.

There are two ways to alleviate this issue. One is to switch to a canard configuration (e.g., Burt Rutan's LongEZ)—in this case the stabilizer will carry an upward lift. But stall characteristics and stability limitations limited the widespread adoption of this configuration. Another approach is to carry upward lift on a conventional stabilizer, and sacrifice longitudinal stability—an approach only doable with fly-by-wire technology and high-speed stability-augmenting control systems (e.g., F16). This is not (yet…) an option for GA airplanes.

Winglets

Winglets seems to sprout on wingtips everywhere, from airliners to gliders. Why?



The main goal of winglets is to reduce induced drag. Only a detailed flow analysis can provide exact gains, but the following qualitative analysis gives an indication on how a gain is possible.

Consider the flow around the wingtip illustrated in the top figure. It implies an inbound (wing tip to wing root) flow component $\boldsymbol{v_i}$ on the upper side of the wing. Looking from the top (middle figure), this component combines with the free-stream velocity $\boldsymbol{V}$ to form a velocity relative to the winglet that is offset by an angle α from $\boldsymbol{V}$. The lift generated by the winglet is perpendicular to the relative velocity, and as shown on the bottom figure, has a component (in red) that *pulls* the wing forward—a negative drag.

This gain in induced drag comes however at with an associated cost—the increased parasite drag due to the additional surface area.

Pros and cons of winglets

- Improves low speed drag, where induced drag is dominant
- Increases drag at high speeds, where parasite drag is dominant

Why winglets rather than span extension (which are better at reducing induced drag)?

- Structural considerations
  - winglets have a smaller impact on bending moments and shear stresses than wing extensions
- Wing span considerations
  - ICAO classifies airplanes in discrete classes based on wingspan. Moving from one class to a higher one increases ramp costs for the airline. Winglets allow decreasing induced drag without increasing wingspan.

Glider case study (http://www.mandhsoaring.com/why%20winglets/wl-it.pdf)

- Gliders are divided in classes based on wingspan—15m class, 18m class, 20 m class
- Winglets provide the potential of reduced drag (at least at low speed) within the wingspan constraint

Glider Case Study

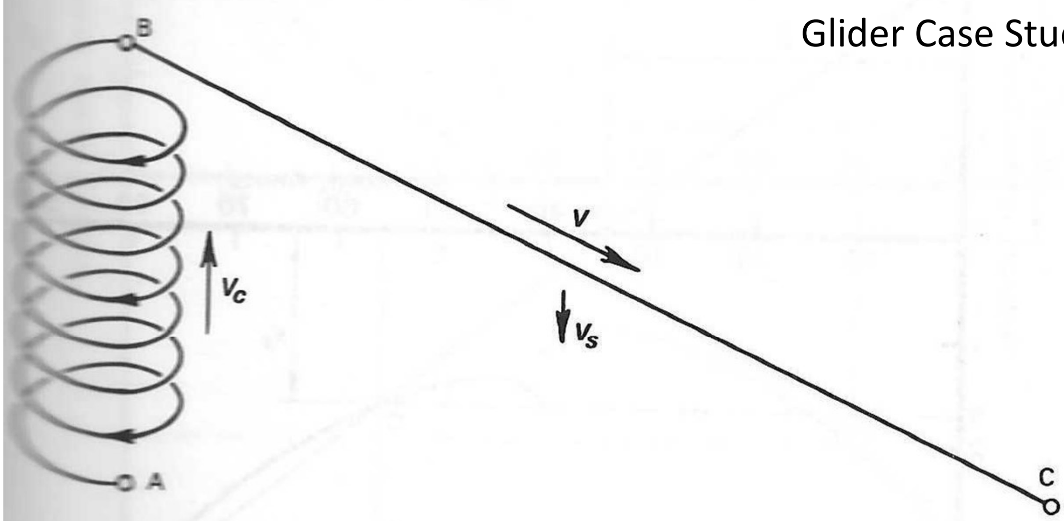


Fig 16.11 Diagram of an idealised cross-country flight

Glider competitions consist of races around a fixed course—the competitor who completes the course in the least time wins.

A typical flight consists of climbs in thermals (low airspeed corresponding to minimum sink, induced drag is predominant), followed by a fast glide (the optimal speed is often considerably higher than best L/D speed) where parasite drag is dominant. The stronger the thermals, the faster the optimal glide speed.

So, while winglets will definitely help during the climb phase, they have the potential to be detrimental during the glide phase, and the designer will have to carefully balance the advantages/inconvenients.

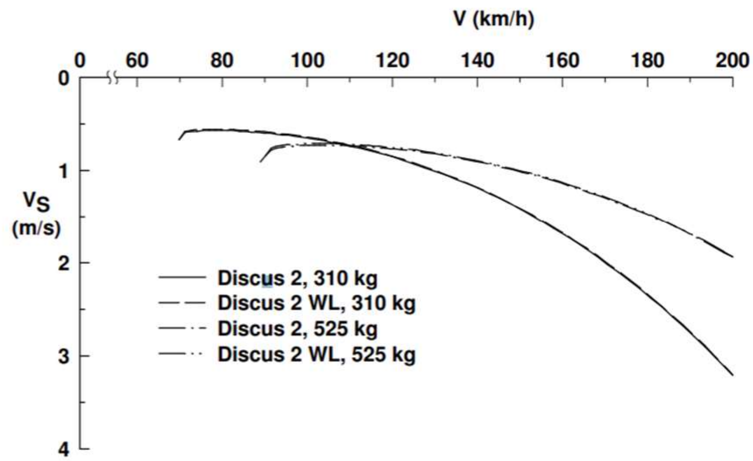


Fig. 8 Predicted straight flight polars of unballasted and ballasted Discus 2, with and without winglets.

The predicted impact of winglets on a **Schempp-Hirth Discus-2** is clearly rather marginal… (This not entirely unexpected, since a high-performance glider has a large aspect-ratio and consequently a low induced drag relative to a typical airplane, and it is difficult to improve a lot when starting from an already very efficient design.)

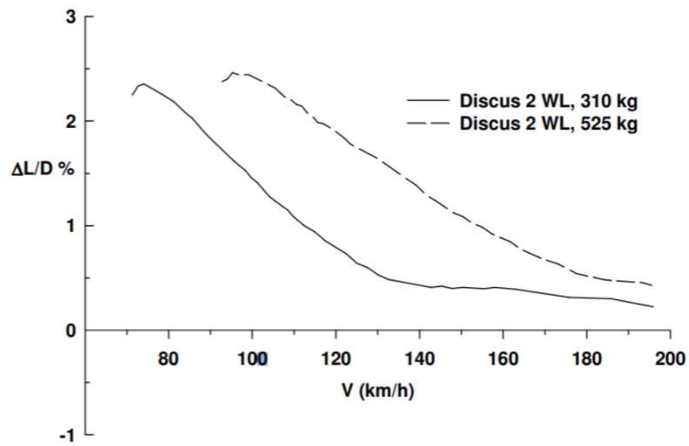


Fig. 10 Percentage gain in predicted lift-to-drag ratios due to winglets for unballasted and ballasted Discus 2.

Looking at the relative increase in L/D ratio, we see an improvement of the order of 2-3% due to winglets. This is confirmed by experimental data.

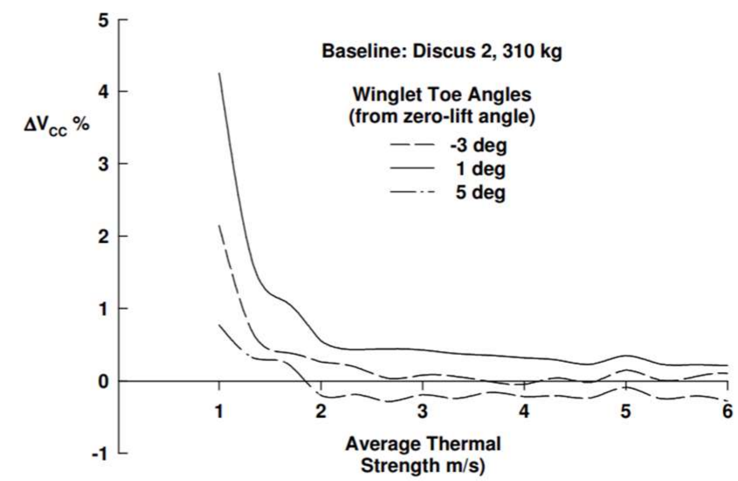


Fig. 12 Percentage change in predicted average cross-country speed as it depends on winglet toe angle for an unballasted Discus 2. Toe angles are measured relative to the zero-lift angle of attack.

Finally, the most important characteristic from the competitor’s perspective is the average cross-country speed, which is a function of the average thermal strength. The best design provides an increase of the order of 4% for weak thermals, decreasing to less than 0.5% for thermal strength greater than 2 m/s. While small, this advantage can be decisive at the highest level of competition, but are probably not very relevant for recreational flying (except maybe in the case of a day with very weak thermals, where the presence of winglets can make the difference between staying up and going down!).